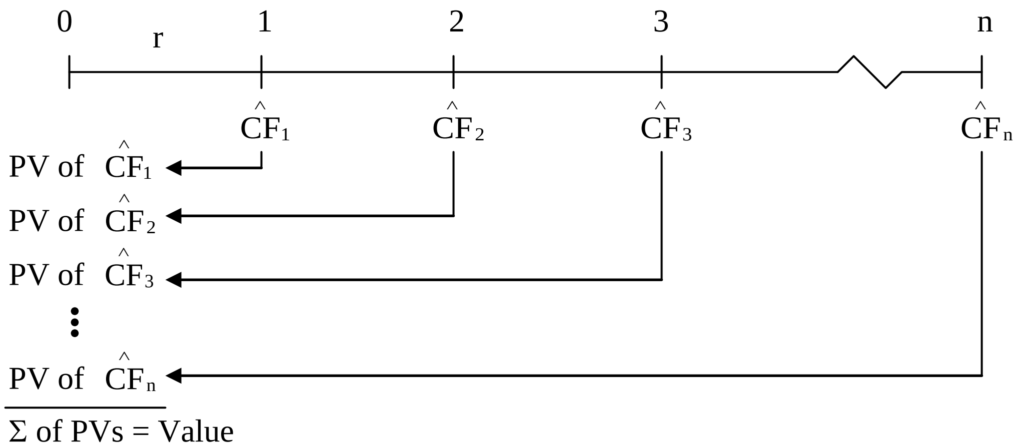
VALUATION (BONDS AND STOCK)

The general concept of valuation is very simple—the current value of any asset is the present value of the future cash flows it is expected to generate. It makes sense that you are willing to pay (invest) some amount today to receive future benefits (cash flows). As a result, the market price of an asset is the amount you must pay today to receive the cash flows the asset is expected to generate in the future. You should not be willing to pay the asset's market price if you can create the same future cash flow stream yourself by investing a lower amount in other investments—for example, a savings account.

- Basic Valuation—if the expected future cash flows and the opportunity cost of an investment can be determined, then the value of the investment can be computed—the value is simply the present value of the future cash flows generated by the investment, which can be depicted on a cash flow time line as follows:



According to the cash flow time line, the equation to compute the value of an asset is:

$$\text{Asset value} = \frac{\hat{CF}_1}{(1+r)^1} + \frac{\hat{CF}_2}{(1+r)^2} + \cdots + \frac{\hat{CF}_n}{(1+r)^n} = \sum_{t=1}^{n} \frac{\hat{CF}_t}{(1+r)^t}$$

where $\hat{CF}_t$ represents the cash flow expected to be generated by the investment in Period t and r is the rate of return investors require to hold this type of investment.

Bonds (Debt)—Characteristics and Valuation (Chapter 6)

- Debt Characteristics—debt is a loan
  - Principal Value, Face Value, Maturity Value, and Par Value—all of these terms refer to the amount that is borrowed, thus the amount that has to be repaid, generally at maturity.
  - Interest Payments—interest generally is paid based on market rates. Some debt does not specifically pay interest; rather the debt is sold at a discount—that is, at a price that is below its principal value—and the investor receives the principal value when the debt matures.

  - Maturity Date—the date the debt matures; the date by which all the principal has to be repaid.
  - Priority to Assets and Earnings—when earnings or liquidation proceeds are distributed, debt holders have priority over equity holders.
  - Control of the Firm—debt holders do not have voting rights.

- Types of Debt—many types of debt exist; debt is classified based on time to maturity when issued, the purpose of the debt, issuers of debt, investors who purchase debt, and so forth.
  - Short-Term Debt—debt that matures in one year or less when originally issued. Examples of such debt include: Treasury bills, repurchase agreements, commercial paper, and money market mutual funds.
  - Long-Term Debt—debt that with maturities greater than one year when issued.
    - Term loans—generally a bank loan that requires the firm to make a series of payments that consist of interest and principal (amortized loan)
    - Bonds—a bond is long-term debt; generally interest is paid throughout the period the bond is outstanding and the entire principal (amount borrowed) is paid back at maturity. Interest payments are based on the bond's coupon interest rate, which is the rate that is applied to the principal amount to determine the dollar interest that is paid. For example, if the coupon rate is 10 percent on a $1,000 face value bond, then $100 interest is paid each year. Interest often is paid semiannually, which means $50 would be paid every six months in this case.
      - Government bonds—bonds issued by federal, state, and local governments. Bonds issued by state and local governments are called municipal bonds, or munis for short.
      - Corporate bonds—bonds issued by corporations
      - Mortgage bonds—bonds that have real (tangible) assets pledged as collateral
      - Debenture—an unsecured bond; subordinated debentures represent debt that ranks below other debt with respect to claims on the firm's assets
      - Income bonds—pay interest only when the firm generates sufficient income
      - Putable bonds—can be turned in and exchanged for cash by investors if the firm takes a particular action
      - Indexed, or purchasing power, bonds—interest payments are pegged to some inflation index, perhaps the CPI
      - Floating-rate bonds—interest is pegged to some market index, perhaps the rate on T-bills
      - Zero coupon bonds—the coupon rate of interest is zero, so no interest is paid; the market prices of these bonds are discounted below the maturity value
      - Junk bonds—high-risk, high-yield bonds

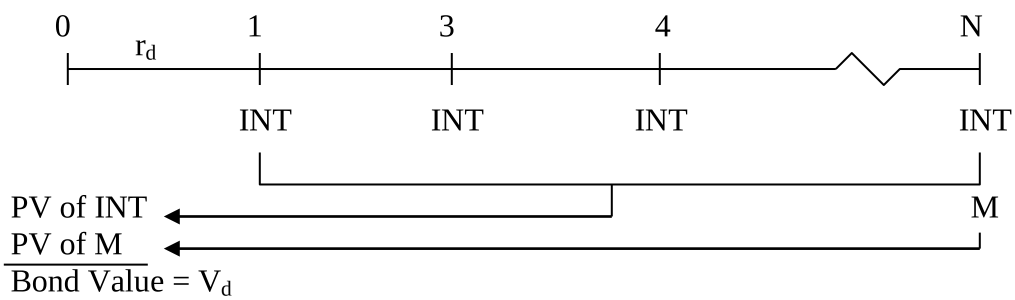
- Bond Contract Features
  - Bond Indenture—the bond contract that specifies maturity date, principal amount, coupon interest, and other features of the bond
  - Call Provision—a feature that allows the issuer of a bond (borrower) to call it in for repayment prior to maturity; not all bonds have call provisions.
  - Sinking Fund—a provision to repay the principal amount over a period of time
  - Convertible feature—a convertible bond can be converted into the firm's common stock at the option of the investors

- <u>Bond Ratings</u>—bond ratings provide an indication of the default risk associated with a particular bond. Because bonds with poorer ratings are considered riskier, the yields on such bonds are higher than bonds with better ratings. Some organizations (institutional investors) cannot invest in bonds with low/poor ratings.

- <u>New issues</u>—the coupon rates on bonds will be close to the interest rates that exist in the market when the bonds are issued; *seasoned issues*, which are bonds that have been traded in the financial markets for a while, could have coupon rates that differ significantly from current market rates because the market rates differed significantly at the time they were issued—for example, some bonds traded in the financial markets perhaps were originally issued 10 to 20 years ago when interest rates were different than today.

- <u>Foreign Debt Instruments</u>
  - Foreign debt—debt sold by foreign issuers
  - Eurodebt—debt sold in a country other than the one in the currency in which it is denominated

- <u>Valuation of Bonds</u>—the coupon rate specifies the amount of interest that is paid each year, and the market value of a bond changes as market interest rates change.
  - The basic bond valuation model—the future cash flows associated with a bond include interest payments and the repayment of the amount borrowed. The cash flows associated with a bond are depicted as follows:



where r_d is the rate investors require on bonds with similar risk, N is the number of periods until maturity, INT is the *dollar* interest paid each period, and M is the maturity, or face, value of the bond.

Based on the cash flow time line given here, the computation for the value of a bond is:

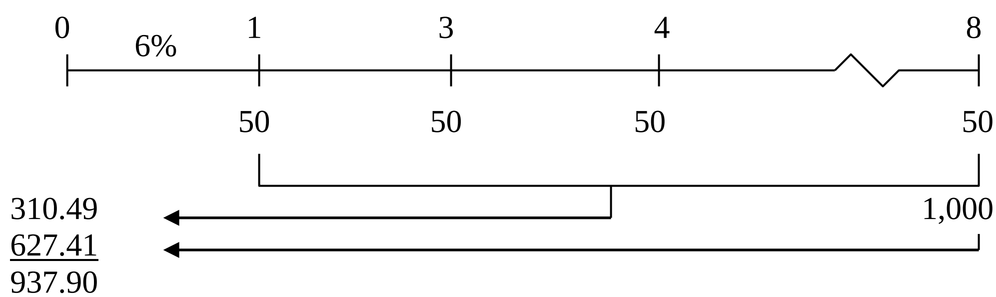
$$\begin{matrix}\text{Bond}\\ \text{Value}\end{matrix} = V_d = \frac{INT}{(1+r_d)^1} + \frac{INT}{(1+r_d)^2} + \frac{INT}{(1+r_d)^3} + \cdots + \frac{INT}{(1+r_d)^N} + \frac{M}{(1+r_d)^N}$$

$$= \sum_{t=1}^{N} \frac{INT}{(1+r_d)^t} + \frac{M}{(1+r_d)^N} = INT\left[\frac{1-\frac{1}{(1+r_d)^N}}{r_d}\right] + M\left[\frac{1}{(1+r_d)^N}\right]$$

Suppose a friend of yours is interested in investing in a corporate bond that has a face value of $1,000, a coupon rate equal to 5 percent, and eight years remaining until maturity. Generally, we would refer to this bond as an 8-year, 5 percent bond, and, unless stated otherwise, we assume the face value is $1,000. Help your friend out by computing the price that should be paid for this bond if the market return on bonds with similar risk is equal to 6 percent. Assume interest is paid annually.

Using the approaches described in the time value of money (TVM) notes, the solution is as follows:

- ***Time Line Solution:***



- ***Equation (Numerical) Solution****:* Using the relationships given earlier, we have the following situation:

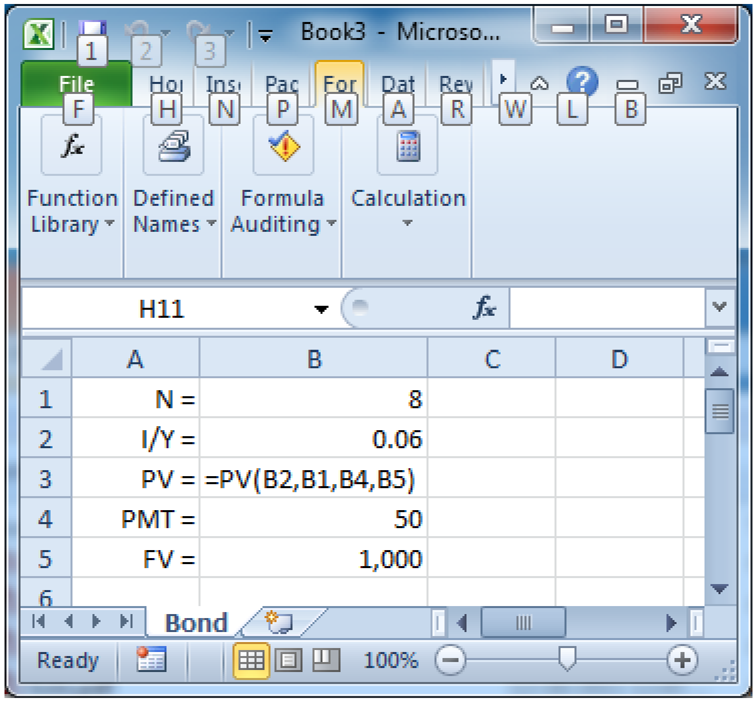
$$V_d = INT\left[\frac{1-\frac{1}{(1+r_d)^N}}{r_d}\right] + M\left[\frac{1}{(1+r_d)^N}\right]$$

$$= \$50\left[\frac{1-\frac{1}{(1.06)^8}}{0.06}\right] + \$1{,}000\left[\frac{1}{(1.06)^8}\right] = \$50(6.20979) + \$1{,}000(0.62741) = \$937.90$$

- ***Financial Calculator Solution****:*

Inputs:	8	6	?	50	1000
	N	**I/Y**	**PV**	**PMT**	**FV**
Result:			**= -937.90**		

- ***Spreadsheet Solution:*** Using Excel, the current problem can be solved using the PV function described in the time value of money section of the notes, the bond's characteristics can be set up as follows:



Notice that, in this case, you can use the same PV function described in the time value of money (TVM) section of the notes, but now both the PMT and FV cells have values entered.

- Finding Bond Yields (Market Rates): Yield to Maturity and Yield to Call
  - Yield to Maturity (YTM)—the return earned on a bond that is purchased and held until maturity is termed the bond's *yield to maturity, YTM*. The YTM associated with a bond basically represents the average rate of return that is earned on the bond from now until it matures. Consider the fact that we can find the market value of a bond by looking in a financial publication such as *The Wall Street Journal*, which also gives information about the bond's maturity and its coupon interest rate. For example, suppose IBM has a 5-year, 10 percent bond outstanding that currently is selling for $1,123. First, let's interpret this information—the bond has five years until it matures, the coupon rate of interest is 10 percent, and its market price is $1,123. The dollar interest paid each year is $100 = 0.10 × $1,000 (at this point we will assume the bond pays interest annually, although interest probably is paid semiannually).

    - ***Equation (Numerical) Solution:*** Plugging this information into the equation we use to compute the value of a bond, we have:

$$\$1{,}123 = \frac{\$100}{(1+r_d)^1} + \frac{\$100}{(1+r_d)^2} + \frac{\$100}{(1+r_d)^3} + \frac{\$100}{(1+r_d)^4} + \frac{\$100}{(1+r_d)^5} + \frac{\$1{,}000}{(1+r_d)^5}$$

$$= 100\left[\frac{1 - \frac{1}{(1+r_d)^5}}{r_d}\right] + 1{,}000\left[\frac{1}{(1+r_d)^5}\right]$$

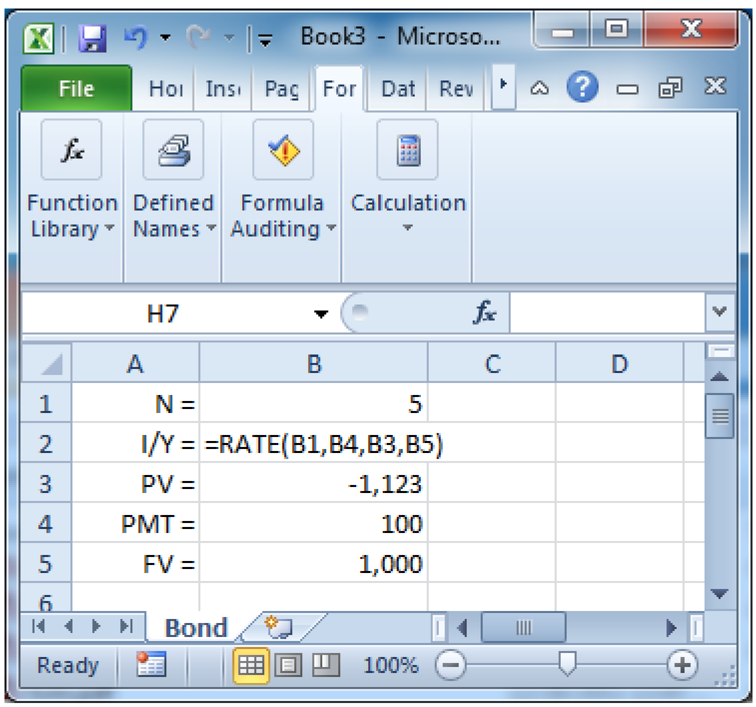
Solving for r_d gives us the YTM for this bond—that is, r_d = YTM. Using the equation, you would have to use a trail-and-error approach to solve for r_d.

- ***Financial Calculator Solution:*** The easiest way to compute r_d is to use the TVM keys on your calculator, which gives the following solution:

Inputs:	5	?	-1,123	100	1,000
	N	**I**	**PV**	**PMT**	**FV**
Result:		**7.0**			

Therefore, the yield to maturity for this bond is 7.0 percent. Notice that the YTM, 7.0 percent, is much lower that the coupon rate of interest, 10 percent. This is because the bond's market price is $1,123, which is $123 greater than the bond's face value, $1,000.00. The relationship between the coupon rate of interest and the market rate of interest, or yield to maturity, and the price of a bond will be discussed later.

- ***Spreadsheet Solution:*** If you use a spreadsheet to solve for the YTM, you can use the RATE function discussed in the time value of money (TVM) section of the notes. The inputs are: Nper = 5, Pmt = 100, Pv = –1,123, and Fv = 1,000. You can set up the spreadsheet as follows:



Let's consider another bond: Arctic Heating has a 6-year, 11½ percent bond that currently is selling for 80¼, which means the market price of the bond is 80.25 percent of its face value, or \$802.50 = \$1,000 x 0.8050.

- ***Financial Calculator Solution:*** Assuming interest is paid annually, we have the following solution for the YTM of this bond:

Inputs:	6	?	-802.50	115	1,000
	N	**I**	**PV**	**PMT**	**FV**
Result:		**17.00**			

In this case, because the bond is selling for a fairly substantial discount (\$97.50 = \$1,000 - \$802.50), its YTM, 17.00 percent, is much greater than the coupon rate of interest, 11.50 percent.

The YTM on the Arctic Heating bond, 17.00 percent, is much greater than the YTM on the IBM bond, 6.84 percent. Why? The primary reason is risk—Arctic Heating has much greater default risk than IBM.

If you do not have a financial calculator, you can *approximate* a bond's yield to maturity using the following equation:

$$\text{Approximate yield to maturity} = \frac{\text{INT} + \left(\frac{M - V_d}{N}\right)}{\left[\frac{2(V_d) + M}{3}\right]}$$

Solving for the approximate YTM for the original IBM bond and the Arctic Heating (AH) bond, we have:

$$\text{Approximate YTM}_{\text{IBM}} = \frac{\$100 + \left(\frac{\$1{,}000 - \$1{,}123}{5}\right)}{\left[\frac{2(\$1{,}123) + \$1{,}000}{3}\right]} = \frac{\$75.40}{\$1{,}082} = 0.0697 = 6.97\% \approx 7.0\%$$

$$\text{Approximate YTM}_{\text{AH}} = \frac{\$115.00 + \left(\frac{\$1{,}000 - \$802.50}{6}\right)}{\left[\frac{2(\$802.50) + \$1{,}000}{3}\right]} = \frac{\$147.92}{\$868.33} = 0.1703 = 17.03\%$$

As you can see, the approximate YTMs are fairly close to the actual YTMs.

- Yield to Call (YTC)—the yield to call is the same as the yield to maturity, except the return is computed using the call price and the call date of the bond rather than the maturity value and the maturity date of the bond (assuming the bond has a call provision). For example, suppose a firm has a 10-year, 9 percent bond outstanding that currently is selling for \$1,040. The bond is callable in five years for \$1,090 (call price). Interest is paid annually.

- ***Equation (Numerical) Solution:*** Plugging this information into the equation we use to compute the value of a bond, the YTM is:

$$V_d = \frac{INT}{(1+YTM)^1} + \frac{INT}{(1+YTM)^2} + \cdots + \frac{INT + \text{Maturity value}}{(1+YTM)^N}$$

$$\$1{,}040 = \frac{\$90}{(1+YTM)^1} + \frac{\$90}{(1+YTM)^2} + \cdots + \frac{\$90}{(1+YTM)^{10}} + \frac{\$1{,}000}{(1+YTM)^{10}}$$

The YTC is computed as follows:

$$V_d = \frac{INT}{(1+YTC)^1} + \frac{INT}{(1+YTC)^2} + \cdots + \frac{INT}{(1+YTC)^{N_c}} + \frac{\text{Call value}}{(1+YTC)^{N_c}}$$

$$\$1{,}040 = \frac{\$90}{(1+YTC)^1} + \frac{\$90}{(1+YTC)^2} + \frac{\$90}{(1+YTC)^3} + \frac{\$90}{(1+YTC)^4} + \frac{\$90 + \$1{,}090}{(1+YTC)^5}$$

- ***Financial Calculator Solution:***
  - ♦ YTM

Inputs:	10	?	-1,040	90	1,000
	N	**I**	**PV**	**PMT**	**FV**
Result:		**8.39**			

Therefore, the yield to maturity (YTM) for this bond is 8.39 percent.

  - ♦ YTC

Inputs:	5	?	-1,040	90	1,090
	N	**I**	**PV**	**PMT**	**FV**
Result:		**9.45**			

The yield to call (YTC) for this bond is 9.45 percent.

If the bond is called in five years, the company will pay a call price equal to $1,090 to retire the debt. In this case, investors who buy the bond today and hold it until the call date will earn an average return equal to 9.45 percent. If the bond is not called and investors hold it until maturity, their average return will be 8.39 percent.

- Interest Rates and Bond Values—the market values of bonds are inversely related to market interest rates—that is, when interest rates increase, the values of bonds decrease. This relationship exists because investors who buy bonds (or other investments) expect to earn the market rate of return. As a result, if market rates increase, investors are willing to pay lower prices to purchase bonds so that they can earn the higher returns. Remember that the coupon interest and maturity value of a bond do not change, even when market conditions do change, because they are fixed contractually (in the indenture). So, because the payments made by the bond issuer to bondholders do not change, a change in the market price of a bond effectively adjusts the bond's market return.

To illustrate the relationship between the market value of a bond and its market return (YTM), suppose that there exists a 10-year, 6 percent bond with a $1,000 face value. The following table shows the relationships between market rates and coupon rates (= INT/M) on this bond at three different market rates (YTMs):

				Current example: N = 10 yrs, INT = $60, M = 1,000	
Relationship of Interest Rates			Selling Price	r_d = YTM	Value = V_d =
Market rate	=	Coupon rate	par	6%	$1,000.00
Market rate	>	Coupon rate	discount	10%	$ 754.22
Market rate	<	Coupon rate	premium	4%	$1,162.22

This table indicates that when the market rate is 6 percent, the bond will sell at par—that is, $1,000. This makes sense, because bondholders will receive $60 per year for a $1,000 investment, which represents a 6 percent yield. On the other hand, when the market rate is 10 percent, the bond will sell for $754.22, which is lower than the face value—that is, a discount. If investors buy the bond for $754.22, they will receive $60 per year and $1,000 at maturity, which is an average annual return that is approximately 10 percent.

Suppose you bought this bond for $1,000 when it was originally issued, and you now want to sell it when the market rate (YTM) is 10 percent. Because you paid $1,000, you want to sell the bond for $1,000 so that you don't lose any of your original investment. Would investors be willing to pay $1,000 for the bond today if the market rate on similar bonds is 10 percent? The answer is NO! Any investor who buys your bond for $1,000 will be paid $60 interest per year, which will generate a 6 percent return. Why would any investor pay you $1,000 for your bond to earn a 6 percent return when he or she could buy a similar bond from someone else and earn a 10 percent return? Rational investors would not want to pay $1,000 for your bond, so you have to lower the price to an amount that would equate the YTM that is earned by the investor who buys your bond to the 10 percent opportunity cost that currently exists in the financial market. Thus, you would be able to sell your bond for only $754.22.

Notice that when the market rate is below the bond's coupon rate—for example, 4 percent—you can sell your bond for a premium—$1,162.22 in this case. If you offered your bond for $1,000, every investor would want to purchase it, because the bond's yield would be 6 percent at the same time the market rate is 4 percent. As a result, you can actually sell your bond for a price that is higher than $1,000. If you sell the bond for $1,162.22, the investor who buys it will expect to earn a YTM equal to approximately 4 percent, which is the current market return.

- Change in Bond Values Over Time—No matter the current value of a bond, it must sell for its face value at the maturity date (assuming the issuer does not default) because this amount is repaid at maturity. Therefore, all else equal, the value of the bond that is selling for a discount (premium) must increase (decrease) as the term to maturity decreases—that is, the bond gets closer to maturity—because its value at maturity must equal $1,000. To see that this is the case, let's compute the price of a bond, which has the following characteristics, each year as it approaches its maturity.

Face value	$1,000
Years to maturity	5
Interest	$60
YTM = r_d	8%

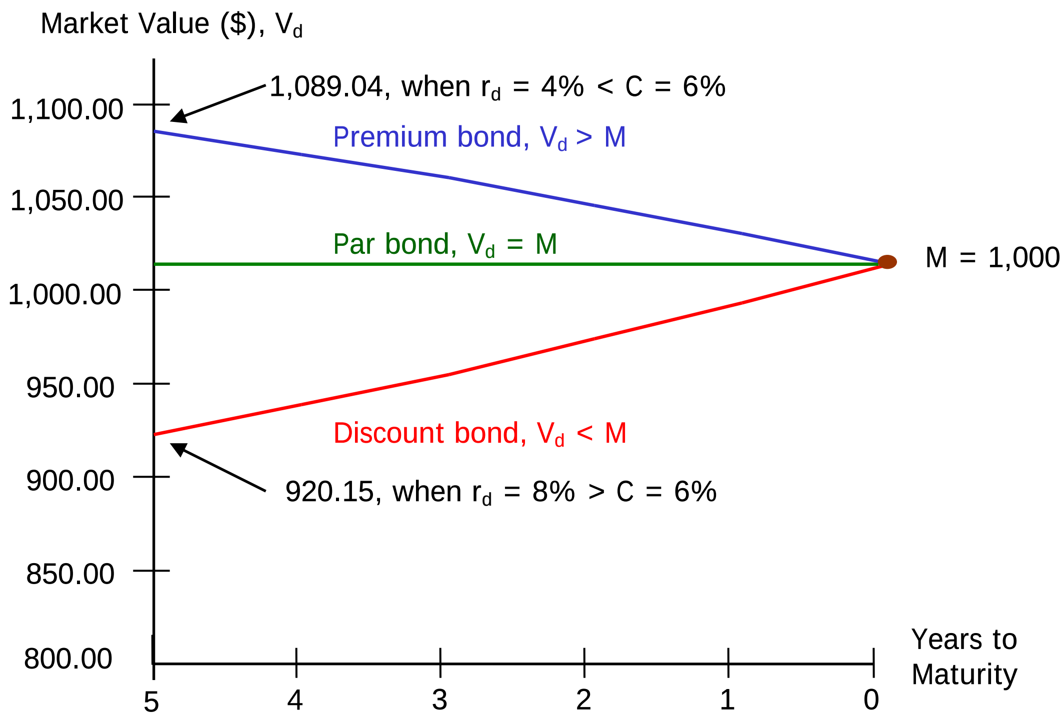
Using your calculator, simply enter N = 5, I/Y = 8, PMT = 60, and FV =1,000, and then solve for PV. The following table shows what happens to the value of the bond as the maturity date gets closer. To check these figures, just decrease the value you enter for N by one for each year that passes and then recompute the PV.

Years to Maturity	End of year Bond Value	Percent Change in Value—Capital Gain	+	Current Yield	=	Total Return
5	$ 920.15					
4	933.76	1.48%		6.52%		8.00%
3	948.46	1.57		6.43		8.00
2	964.33	1.67		6.33		8.00
1	981.48	1.78		6.22		8.00
0	1,000.00	1.89		6.11		8.00

In the table, the *capital gains yield* represents the change—increase in this example—in the value of the bond from one year to the next, which is computed as $[(V_{d1} – V_{d0})/V_{d0}] –1.0$. For example, 1.48% = ($933.76 – $920.15)/$920.15. The *current yield* is computed by dividing the dollar interest received during the year by the value of the bond at the *beginning* of the year—for example, 6.52% = $60/$920.15, 6.43% = $60/$933.76, and so forth.

Notice that, in each year, the total return is 8 percent. Remember that we valued the bond using r_d = 8%, which indicates that investors demand an 8 percent return to purchase this bond. Therefore, everything else equal, each year, investors will expect an 8 percent return. What do you think would happen if the bond paid a lower return, say, 7 percent, even though investors demanded an 8 percent return? Investors would no longer find the bond attractive, thus they would quit investing in it, which would cause the price to drop and the return to increase until it reached 8 percent. Think about what would happen if the bond paid a return greater than 8 percent. Answer this question by asking yourself whether you would want to own such a bond.

The following graph shows the values of the bond in our illustration as it approaches its maturity date for three different scenarios: (1) the bond is currently selling at a premium—that is, V_d = $1,089.04, (2) the bond is selling at its par value—that is, V_d = $1,000, and (3) the bond is selling for a discount—that is, V_d = $920.15



- <u>Bond Values with Semiannual Compounding</u>—most bonds pay interest on a semiannual basis rather than annually like we have assumed to this point. For such bonds, we need to adjust the discount rate, r_d, and the number of periods just like we did for time value of money problems with multiple compounding in the year. For example, let's assume that the bond we previously evaluated that has a $1,000 face value, five years remaining until maturity, and a coupon interest rate equal to 6 percent pays interest semiannually. If investors demand an 8 percent return (annual) to invest in similar risk bonds, then the price of this bond should be:

$$V_d = \$30\left[\frac{1-\frac{1}{(1.04)^{10}}}{0.04}\right] + \$1{,}000\left[\frac{1}{(1.04)^{10}}\right]$$

$$= \$30(8.110896) + \$1{,}000(0.675564) = \$918.89$$

According to this computation, the bond pays $30 every six months (semiannually), the investor has the opportunity to invest at 4 percent interest every six months, and there are 10 6-month periods during the remaining life of the bond. Thus, if you invest in this bond, you would receive a $30 annuity paid twice a year for the next five years.

- <u>Interest Rate Risk on a Bond</u>—interest rates change constantly in the financial markets, thus investors face *interest rate risk* continuously with their investments. Interest rate changes affect investors who hold bonds in two ways—when interest rates change, (1) the market value of the bond changes and (2) so does the rate at which the interest received by investors can be reinvested. The following table shows what would happen to the value of the bond we examined earlier ($60 interest payment annually and five years until maturity) if interest rates *immediately* after the bond was purchased are as follows:

Market Rate, r_d	Bond Value, V_d	
4%	$1,089.04	
6	1,000.00	
8	**920.15**	**–** current market value
10	848.37	
12	783.71	

Notice that the value of the bond is inversely related to the market rate—if the rate increases, the value decreases. The reason for this is because investors receive a fixed amount of future cash flows (interest) when they buy bonds like the one we are examining here. If the market rates change, the bond's value changes such that if an investor purchases the bond after the rate change, he or she will earn a YTM equal to the market rate of return. Consider what would happen if you purchased the bond when the market rate was 8 percent, thus you paid $920.15. As soon as the purchase was completed the market rate jumped to 10 percent, so you decide to sell the bond so that you can invest in a different bond that earns 10 percent rather than 8 percent. Because you paid $920.15 for the bond and only held it for a short time, you might think you can sell it for the same price. Who do you think would be willing to buy the bond? The answer is *NOBODY*, because investors can now earn 10 percent on similar risk bonds and if they purchased your bond for $920.15 they would only earn 8 percent. As a result, you must adjust the price of your bond so that it now generates a 10 percent return—that is, you would have to drop the price to $848.37. Because the *price* of the bond changes each time market interest rates change, we have what is termed *interest rate* ***price*** *risk.*

Given the preceding discussion, you might think that when interest rates increase, bondholders are unhappy because the prices of their bonds decrease. But, although the price of the bond in our example would drop to $848.37 if interest rates increased from 8 percent to 10 percent, you would now be able to invest the interest payment received from the bond every year at 10 percent compounded annually rather than 8 percent. In this case, we have what is termed *interest rate* ***reinvestment*** *risk*. And, as you can tell, its effect is opposite that of interest rate price risk. That is, if market interest rates increase (decrease) the prices of bonds decrease (increase), but investors are able to reinvest any income received from the bonds at the higher (lower) rates.

Stocks (Equity)—Characteristics and Valuation (Chapter 7)

- Preferred Stock—generally pays a constant dividend similar to a bond, but the firm cannot be forced into bankruptcy if the dividend is not paid, which is similar to common stock—often called a hybrid security
  - Par value—(1) the amount that is paid to stockholders in the event the company is liquidated, assuming that the funds are available, and (2) the dividend generally is stated as a percent of the par value.
  - Cumulative dividends—a feature that requires the firm to pay preferred dividends that were not paid in previous periods before any common stock dividends are paid.
  - Maturity—does not have a specific maturity.
  - Priority to assets and earnings—preferred dividends are paid after interest on debt is paid, but before common stock dividends are paid; in the event of liquidation of the company, preferred stockholders are paid after debt obligations are repaid but before common stockholders receive any liquidation proceeds.
  - Control of the firm—preferred stockholders do not having voting rights; the exception is when preferred dividends are not paid for a particular (extended) period of time.
  - Convertibility—like bonds, preferred stock can contain a provision that permits investors to convert into the common stock of the firm.
  - Other provisions—(1) call premium—the amount that the firm has to pay if it calls in preferred stock, (2) sinking fund—used to repurchase and retire some preferred stock at periodic intervals, and (3) participating preferred—receives a stated minimum dividend and then participates with common stockholders in the distribution of earnings in excess of some stated amount.

- Common Stock—common stockholders are the "owners" of the firm
  - Par value—common stock does not have to have a par value; the par value of common stock refers to the minimum amount for which a stockholder is personally liable, which means that investors who purchase the stock for more than its par value cannot lose any more money than what they invest in the stock.
  - Dividends—dividends are not guaranteed; they do not have to be paid. However, any income left after paying interest on debt and preferred stock dividends "belongs" to common stockholders, and this amount is provided to stockholders as dividend payments, reinvestment in the firm (through retained earnings), or both.
  - Maturity—there is no specific maturity
  - Priority to assets and earnings—common stockholders are "last in line" when it comes to distribution of earnings and liquidation proceeds; common stockholders are paid after (1) debt holders, and (2) preferred stockholders.
  - Control of the firm (voting rights)—common stockholders have the right to vote to elect the firm's board of directors, who then appoint the officers. Many investors assign their voting privileges to another person; this is called a proxy.
  - Preemptive right—some common stock allows existing stockholders the right to purchase new issues of stock in the proportion that they own prior to a new issue; this permits stockholders to maintain the same percentage control of the firm after the stock is issued.
  - Types of common stock—many firms issue "classified" common stock, such as Class A, Class

B, and so forth; in many cases, one class is voting stock and another class is non-voting

- Equity Instruments in International Markets
  - American Depository Receipts (ADR)—certificates created by financial organizations that represent ownership in foreign stock; investors own the certificates, not the stock; the value of the certificate changes as the value of the underlying stock changes.
  - Foreign Equity (Stock)
    - Euro stock—stock traded in countries other than the home country of the company
    - Yankee stock—stock of a foreign company that is traded in the United States

- Stock Valuation—Dividend Discount Model (DDM)—when we value stock, we use the same approach described for valuing bonds—that is, find the present value of all the cash flows expected to be received from the stock in the future. Thus, in equation form, the value of a share of stock can be written:

$$\begin{array}{c}\text{Value of}\\ \text{stock}\end{array} = V_s = \hat{P}_0 = \text{PV of expected future dividends}$$

$$= \frac{\hat{D}_1}{(1+r_s)^1} + \frac{\hat{D}_2}{(1+r_s)^2} + \cdots + \frac{\hat{D}_\infty}{(1+r_s)^\infty}$$

where $\hat{D}_t$ represents the dividend expected in Period t and r_s is the rate of return investors require to invest in similar risk equity investments. We usually apply the above equation to three different general situations: (1) firms that pay the same dollar amount of dividends—that is, the dividend remains constant such that there is no growth in dividends; (2) firms that pay dividends that grow at a normal, or constant, rate because the firm grows at the same rate—that is, the rate of growth is the same each year; and (3) firms that pay dividends that have different growth rates—that is, the rate of growth is nonconstant.

  - *Valuing stocks with zero growth*—a *zero-growth* stock is one where all future dividends are expected to be the same, such that $\hat{D}_1 = \hat{D}_2 = \cdots = \hat{D}_\infty = D$, and

$$\hat{P}_0 = \frac{D}{(1+r_s)^1} + \frac{D}{(1+r_s)^2} + \cdots + \frac{D}{(1+r_s)^\infty} = \frac{D}{r_s}$$

Because a zero-growth stock has dividend payments that represent a perpetuity, as you can see from the bottom form of the equation, finding the value of such stocks is very simple. For example, if would like to invest in a stock that promises to pay a $6.00 dividend every year, beginning in one year and continuing until the firm stops business, which is never expected to occur, and the required return on similar risk investments is 12 percent, the market value of the stock should be $50 = $6/0.12. (This could be a preferred stock.)

Generally we can find the market value of the stock in such financial sources as *The Wall Street Journal*. Thus, often we use this information to determine the rate of return investors require when investing in similar types of investments. For example, suppose that a stock that pays a constant dividend equal to \$3 per share currently is selling for \$20 per share. We know the following must exist:

$$\$20 = \frac{\$3}{r_s}\text{, which means that } r_s = \frac{\$3}{\$20} = 0.15 = 15.0\%$$

As a result, if a stock pays a constant dollar dividend, we can determine r_s as follows:

$$r_s = \frac{D}{P_0}$$

- *Valuing stocks with normal, or constant, growth*—if a firm has normal, or constant, growth, then the dividend it pays is expected to grow at a constant rate each year. This means that $\hat{D}_1 = D_0(1+g)^1$, $\hat{D}_2 = D_0(1+g)^2$, and so forth (g represents the constant growth rate). Thus, to find $\hat{P}_0$, we have the following:

$$\begin{aligned}\hat{P}_0 &= \frac{D_0(1+g)^1}{(1+r_s)^1} + \frac{D_0(1+g)^2}{(1+r_s)^2} + \cdots + \frac{D_0(1+g)^\infty}{(1+r_s)^\infty} \\ &= \frac{D_0(1+g)}{r_s - g} = \frac{\hat{D}_1}{r_s - g}\end{aligned}$$

The form of the equation given in the last line is an algebraic simplification that results because the growth rate, g, is constant.

Consider a company that just paid a \$2 dividend. The company's dividend has grown at a rate equal to 4 percent each year for the last 20 years, a pattern that is expected to continue for the next 100 years. If the required rate of return on similar risk investments is 12 percent, what is the market value of the firm's stock? This firm's dividend is expected to grow at 4 percent each year for the next 100 years, so the stream of dividends received by stockholders will look like this:

Year	Dividend	Computation, g = 4%	PV of $\hat{D}_t$ @ 12%
1	2.080000	$\hat{D}_1 = \$2(1.04)^1$	1.857143
2	2.163200	$\hat{D}_2 = \$2(1.04)^2$	1.724490
3	2.249728	$\hat{D}_3 = \$2(1.04)^3$	1.601312
4	2.339717	$\hat{D}_4 = \$2(1.04)^4$	1.486933
5	2.433306	$\hat{D}_5 = \$2(1.04)^5$	1.380723
•	•	•	•
•	•	•	•
•	•	•	•
10	2.960489	$\hat{D}_{10} = \$2(1.04)^{10}$	0.953198
25	5.331673	$\hat{D}_{25} = \$2(1.04)^{25}$	0.313627
50	14.213367	$\hat{D}_{50} = \$2(1.04)^{50}$	0.049181
75	37.890509	$\hat{D}_{75} = \$2(1.04)^{75}$	0.007712
100	101.009896	$\hat{D}_{100} = \$2(1.04)^{100}$	0.001209
		Total	25.984278

Using a spreadsheet to compute the dividends and the present values of the dividends for 100 years, we find

$$\text{PV of } \sum_{t=1}^{100} \hat{D}_t = \$25.98$$

If we assume the dividends grow at a constant 4 percent *for an infinite period*, we can apply the *constant growth dividend discount model* as follows:

$$\hat{P}_0 = \frac{D_0(1+g)}{r_s - g} = \frac{\hat{D}_1}{r_s - g}$$

$$= \frac{\$2(1.04)}{0.12 - 0.04} = \frac{\$2.08}{0.08} = \$26.00$$

As you can see, this result is \$0.02 greater than the result found for the PV of the dividends that are expected to be paid for the next 100 years. This suggests that the PV of the dividends expected to be paid from Year 101 to Year ∞ is approximately \$0.02.

- *Expected rate of return on a constant growth stock*—if we rearrange the equation given for the constant growth model, we have the following:

$$\hat{r}_s \quad = \quad \frac{\hat{D}_1}{P_0} \quad + \quad g$$

$$\begin{matrix}\text{Expected rate} \\ \text{of return}\end{matrix} = \begin{matrix}\text{Expected} \\ \text{dividend yield}\end{matrix} + \begin{matrix}\text{Expected growth rate,} \\ \text{or capital gains yield}\end{matrix}$$

If we know the market value of the stock, P_0, the most recent dividend payment, D_0, and the rate at which future dividends are expected to grow, g, we can compute the rate of return that investors expect the stock to yield. For example, suppose a company's stock is currently selling for \$50, the latest dividend paid by the firm was \$2, and future dividends are expected to grow at 7 percent. The expected rate of return for this stock is

$$\hat{r}_s = \frac{\$2.00(1.07)}{\$50} + 0.07 = 0.0428 + 0.07 = 0.1128 = 11.28\%$$

In this situation, P_0 = \$50 and $\hat{D}_1$ = \$2.14 = \$2.00(1.07).

In one year, the price of the stock is expected to be:

$$\hat{P}_1 = \frac{\hat{D}_2}{(1+r_s)^1} + \frac{\hat{D}_3}{(1+r_s)^2} + \cdots + \frac{\hat{D}_\infty}{(1+r_s)^{\infty-1}} = \frac{\hat{D}_2}{r_s - g}$$

$$= \frac{\$2.14(1.07)}{0.1128 - 0.07} = \frac{\$2.2898}{0.0428} = \$53.50$$

Notice that in one year the value of the stock is the present value of the dividends expected to be paid for the remaining life of the firm, which is $\hat{D}_2, \hat{D}_3, \cdots, \hat{D}_\infty$.

Because the value of the stock is expected to increase to \$53.50 one year from now, investors in this stock expect to earn a capital gains yield equal to:

$$\begin{matrix}\text{Capital} \\ \text{gains yield}\end{matrix} = \frac{\text{Ending value - Beginning value}}{\text{Beginning value}} = \frac{\hat{P}_1 - P_0}{P_0}$$

$$= \frac{\$53.50 - \$50.00}{\$50.00} = 0.07 = 7.0\%$$

The dividend yield investors expect equals:

$$\begin{matrix}\text{Dividend}\\ \text{yield}\end{matrix} = \frac{\hat{D}_1}{P_0} = \frac{\$2.14}{\$50} = 0.0428 = 4.28\%$$

Thus, the total expected rate of return for the stock is

$$\hat{r}_s = 7.0\% + 4.28\% = 11.28\%$$

which is the same result we found earlier.

- *Valuing stocks with nonconstant growth*—most firms do not growth at constant rates each year—that is, nonconstant growth exists. For such companies, because growth is not constant, we cannot apply the constant growth model. This suggests that we have to use the technique discussed in the time value of money section of the notes to find the present value of an uneven cash flow stream. Fortunately, however, we can simplify the computation by assuming that a firm that currently experiences nonconstant growth will begin constant growth at some future date. Consider, for example, a company that expects its growth during the next five years to be 18 percent, 16 percent, –6 percent, 10 percent, 8 percent and then growth is expected to level off to 6 percent, where it will remain from that point on. In this case, the firm has nonconstant growth for five years and then constant growth from Year 6 until Year ∞. Think about how to apply your knowledge of present value techniques and the techniques discussed in earlier to determine the value of the stock. In general terms, we would have to compute the value as follows:

$$\hat{P}_0 = \frac{\hat{D}_1}{(1+r_s)^1} + \frac{\hat{D}_2}{(1+r_s)^2} + \cdots + \frac{\hat{D}_N + \hat{P}_N}{(1+r_s)^N}$$

$$= \frac{D_0(1+g_1)}{(1+r_s)^1} + \frac{\hat{D}_1(1+g_2)}{(1+r_s)^2} + \cdots + \frac{\hat{D}_{N-1}(1+g_N) + \hat{P}_N}{(1+r_s)^N}$$

where N is the year in which nonconstant growth ends and thus constant growth begins.

According to this equation, we have to determine the dividends for each period in which the growth rate is nonconstant and find the present value of those dividends. We also have to find the price of the stock at some future date—that is, $\hat{P}_N$ —and compute the present value of this amount.

Because nonconstant growth ends in Year N, the dividend in Year N+1 is $\hat{D}_{N+1} = \hat{D}_N(1+g_{norm})$, where g_{norm} is the normal, or constant, growth rate. When nonconstant growth ends (constant growth begins), we can apply the constant growth dividend model to compute the value of the stock at that point:

$$\hat{P}_N = \frac{\hat{D}_N(1+g_{norm})}{r_s - g_{norm}} = \frac{\hat{D}_{N+1}}{r_s - g_{norm}}$$

Using this equation, we can compute $\hat{P}_N$, which represents the present value in Year N of the dividends that are expected to be paid in Year N+1 through Year ∞, by applying the constant growth model in the year in which dividends start to grow at a constant rate. To illustrate, let's continue with the situation that was introduced earlier—a firm expects its dividends to grow at nonconstant rates for the next five years and then to grow at the constant rate of 6 percent. If the firm recently paid a dividend equal to $1.00 and the required rate of return on similar risk investments is 12 percent, what should be the price of the stock? Using this information, the dividends for the next five years are expected to be:

Year	Dividend, $\hat{D}_t$	Computation	PV of $\hat{D}_t$ @ 12%
1	$1.1800	$\hat{D}_1$ = $1.0000(1.18)	$1.0536
2	1.3688	$\hat{D}_2$ = $1.1800(1.16)	1.0912
3	1.2867	$\hat{D}_3$ = $1.3688(0.94)	0.9158
4	1.4153	$\hat{D}_4$ = $1.2867(1.10)	0.8994
5	1.5286	$\hat{D}_5$ = $1.4153(1.08)	0.8674
			$4.8274

After Year 5, the expected growth rate will be 6 percent per year. As a result, in Year 6 through Year 100 the dividends will be:

Year	Dividend, $\hat{D}_t$	Computation, g = 6%	PV of $\hat{D}_t$ @ 12%
6	$1.6203	$\hat{D}_6$ = $1.5286(1.06)^1	$0.8209
7	1.7175	$\hat{D}_7$ = $1.5286(1.06)^2	0.7769
8	1.8205	$\hat{D}_8$ = $1.5286(1.06)^3	0.7353
9	1.9298	$\hat{D}_9$ = $1.5286(1.06)^4	0.6959
10	2.0456	$\hat{D}_{10}$ = $1.5286(1.06)^5	0.6586
⋮	⋮	⋮	⋮
25	4.9023	$\hat{D}_{25}$ = $1.5286(1.06)^{20}	0.2884
50	21.0401	$\hat{D}_{50}$ = $1.5286(1.06)^{45}	0.0728
100	387.5623	$\hat{D}_{100}$ = $1.5286(1.06)^{95}	0.0046
			$15.2412

Because the dividends grow at a constant rate after Year 5, we can apply the constant growth model at that point such that we have the following:

$$\hat{P}_5 = \frac{\hat{D}_6}{r_s - g_n} = \frac{\$1.6203}{0.12 - 0.06} = \$27.00$$

Thus, the value of the dividends expected in Year 6 through Year 4 is $27.00 *at the end of Year 5*. Stated differently, this stock can be sold for $27 five years from today. If we find the present value of the stock price expected in Year 5, we have

$$\text{PV of } \hat{P}_5 = \frac{\$27.00}{(1.12)^5} = \$15.32$$

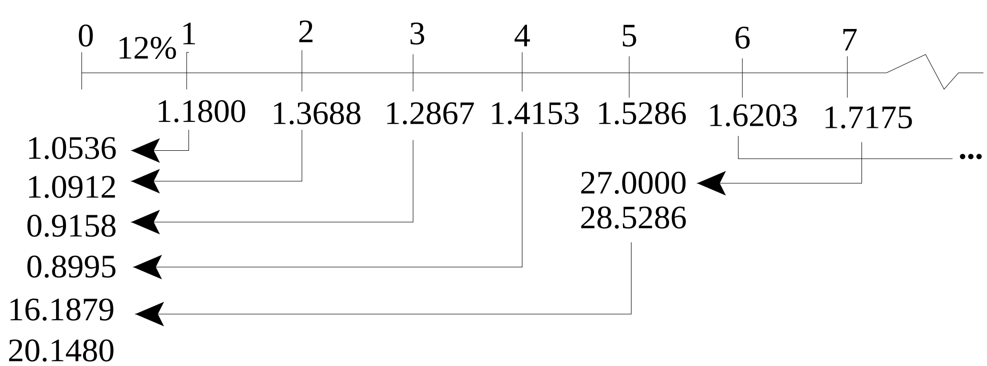
Again, notice that the result is nearly the same as the sum of the present values of the dividends expected in Year 6 through Year 100 shown in the table earlier. Thus, the present value of all the dividends expected in the future is

$$\begin{aligned} P_0 &= \text{PV of } \hat{D}_1 \text{ through } \hat{D}_5 + \text{PV of } \hat{D}_6 \text{ through } \hat{D}_\infty \\ &= \$4.8274 \quad + \quad \$15.32 \quad = \$20.15 \end{aligned}$$

This is the price at which the stock should be selling today.

The key to computing the value of a stock that experiences nonconstant growth is that we assume constant growth occurs at some point in the future—it might start in five years, 50 years, or 100 years. when nonconstant growth ends, we can apply the constant growth model to compute the value of the expected dividends from that point until the firm ceases to exist (i.e., ∞) and then find the present value of that amount. Prior to the point where nonconstant growth occurs, we have to compute the dividend for each year, find the present value of each dividend, and sum the results, just like we did when we encountered an uneven cash flow stream in time value of money problems.

On a cash flow time line, the current situation can be illustrated as follows:



- Other Stock Valuation Techniques—investors generally use more than one method to evaluate stocks
  - *Valuation using P/E ratios*—many investors like to use the price-to-earnings ratio to estimate the value of a stock; remember that the P/E ratio is

$$\text{P / E ratio} = \frac{\text{Market price per share}}{\text{Earnings per share}} = \frac{P_0}{EPS_0}$$

    If we can estimate the firm's earnings per share, a "normal" P/E ratio can be used to estimate the appropriate market value of the stock. For example, if a stock normally has a P/E ratio equal to 12, and analysts estimate that the company's EPS will be $3 per share, then, using the P/E ratio approach the value of the stock would be estimated to be $36 = $3 x 12. In this case, we are saying that the stock normally sells for 12 times its EPS.

  - *Evaluating stocks using the Economic Value Added (EVA) approach*—EVA is based on the concept that earnings should be paid to those who provide funds to the firm—that is, bond holders and equity holders—before the firm's true "economic" earnings can be determined. EVA is the amount by which the firm's value changes after compensating investors for the funds they provide the firm

    EVA = EBIT(1- Tax rate) – [(Invested capital) x (After-tax cost of funds as a percent)]

    To illustrate, suppose we have the following information about a firm:

EBIT	$8.2 million
Total capital = Long-term debt + Equity	$50.0 million
Marginal tax rate	40.0%
Debt/assets ratio	55.0%
Before-tax cost of funds	12.2%
Number of outstanding share of common stock	1.5 million

    EVA = $8.2(1 – 0.4) – [0.122(1 – 0.4)]($50.0) = $4.92 – $3.66 = $1.26

    Thus, the "true" income that the firm generated is $1.26 million. As a result, the firm can pay a common stock dividend equal to $0.84 = ($1.26 million)/(1.5 million shares) before its value is threatened.

- Changes in Stock Prices—we generally consider the stock market to be in equilibrium when the expected rate of return and the required rate of return are equal—that is, $\hat{r}_s$ = r. If the market is not in equilibrium, say, the required return, r_s, is greater than the expected return, $\hat{r}_s$, for a stock, then investors would not want to purchase the stock, which would cause its price to drop and its expected return to increase until $\hat{r}_s = r_s$; the opposite would occur if $r_s < \hat{r}_s$.

- Valuation Summary Questions—You should answer these questions as a summary for the chapter and to help you study for the exam.
    - In simple terms, how is value determined?
        - Valuing bonds
        - Valuing equity
    - What are the components of return in general?
        - What are the components of the return on a bond called? On a stock?
        - What is the yield to maturity (YTM)?
        - What is the relationship between YTM, coupon rate, and market value of a bond?
    - How does the price of a bond change over time? Does the price of a bond change over time even if interest rates do not change? Explain
    - What does it mean for the stock market to be in equilibrium?