## Solutions to TVM Assignment \#1

1. Information given:
i. Shelli will save for 40 years at 8 percent compounded annually, and then retire.
ii. Shelli wants to receive payments equal to $\$ 60,000$ each year, and these payments are expected to last for 25 years.
iii. Upon retirement, Shelli's funds will earn 5 percent compounded annually.

The cash flow time line for Janet is:

| 25 | 26 | 27 | 28 | 65 | 66 | 67 | 89 | 90 | Shelli's age <br> 0 |
| ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :--- |
|  | 1 | 2 | 3 | 40 |  |  |  |  |  |
| Payments |  |  |  |  |  |  |  |  |  |

To solve this problem, you must break it up into two parts. First, determine the amount Shelli needs in her retirement fund when she makes her last contribution to the fund to be able to withdraw $\$ 60,000$ per year for 25 years, beginning on the day that she retires. This is an annuity due. Using a calculator, enter $\mathrm{I} / \mathrm{Y}=5$ (the rate after retirement is 5 percent), $\mathrm{N}=25$, and $\mathrm{PMT}=60,000$; set your calculator to "BGN" mode and compute PV to determine the value of the retirement annuity at retirement is equal to $\$ 887,918.51$. The computation is:

$$
\operatorname{PVA}(\mathrm{DUE})=60,000\left\{\left[\frac{1-\frac{1}{(1.05)^{25}}}{0.05}\right] \times 1.05\right\}=60,000(14.798642)=887,918.51
$$

Thus, at retirement, Shelli needs $\$ 887,918.51$. As a result, the cash flow time line for Shelli today, when she is planning her retirement and wants to determine the amount she needs to contribute to the retirement fund, is as follows:


This is an ordinary annuity because Shelli will not make her first contribution to her retirement fund until one year from today. Using a calculator, enter $\mathrm{N}=40, \mathrm{I} / \mathrm{Y}=8 \%$, and FV $=887,918.51$; compute $\mathrm{PMT}=3,427.51$ each year. The computation is:

$$
\begin{aligned}
\mathrm{FVA} & =\mathrm{PMT}\left[\frac{(1+\mathrm{r})^{\mathrm{n}}-1}{\mathrm{r}}\right]=887,918.51=\mathrm{PMT}\left[\frac{(1.08)^{40}-1}{0.08}\right] \\
887,918.51 & =\mathrm{PMT}(259.056519) \\
\mathrm{PMT} & =\frac{887,918.51}{259.056519}=3,427.51
\end{aligned}
$$

Shelli must contribute $\$ 3,427.51$ each year for 40 years to meet her retirement goals.
2. a. The current cost is $\$ 15,000$ per year, which will escalate at a 3 percent inflation rate:

| College | Current | Years | Inflation | Cash |
| :---: | :---: | :---: | :---: | :---: |
| Year | Cost | from Now | Adjustment | Required |
| 1 | \$15,000 | 3 | $(1.03)^{3}$ | \$16,390.91 |
| 2 | 15,000 | 4 | $(1.03)^{4}$ | 16,882.63 |
| 3 | 15,000 | 5 | $(1.03)^{5}$ | 17,389.11 |
| 4 | 15,000 | 6 | $(1.03)^{6}$ | 17,910.78 |

Now put these costs on a cash flow time line and find the PV at the time Alvin starts college:


General computation:

$$
\mathrm{PV}=\mathrm{FV}\left[\frac{1}{(1+\mathrm{r})^{\mathrm{n}}}\right]=\frac{16,390.91}{(1.08)^{0}}+\frac{16,882.63}{(1.08)^{1}}+\frac{17,389.11}{(1.08)^{2}}+\frac{17,910.78}{(1.08)^{3}}=61,149.48
$$

Thus, the college fund must accumulate $\$ 61,149.48$ by the time Alvin starts college.
b. The key to completing the problem at this point is to realize the series of deposits represent an annuity due. Thus,


Calculator solution: $\mathrm{N}=3, \mathrm{I} / \mathrm{Y}=8, \mathrm{PV}=0, \mathrm{FV}=61,149.48$; set your calculator to BGN mode and solve for $\mathrm{PMT}=-17,440.82$

$$
\mathrm{FVA}=\mathrm{PMT}\left\{\left[\frac{(1+\mathrm{r})^{\mathrm{n}}-1}{\mathrm{r}}\right] \times(1+\mathrm{r})\right\}=61,149.48=\text { PMT }\left\{\left[\frac{(1.08)^{3}-1}{0.08}\right] \times 1.08\right\}
$$

$$
\mathrm{PMT}=\frac{61,149.48}{3.506112}=17,440.82
$$

