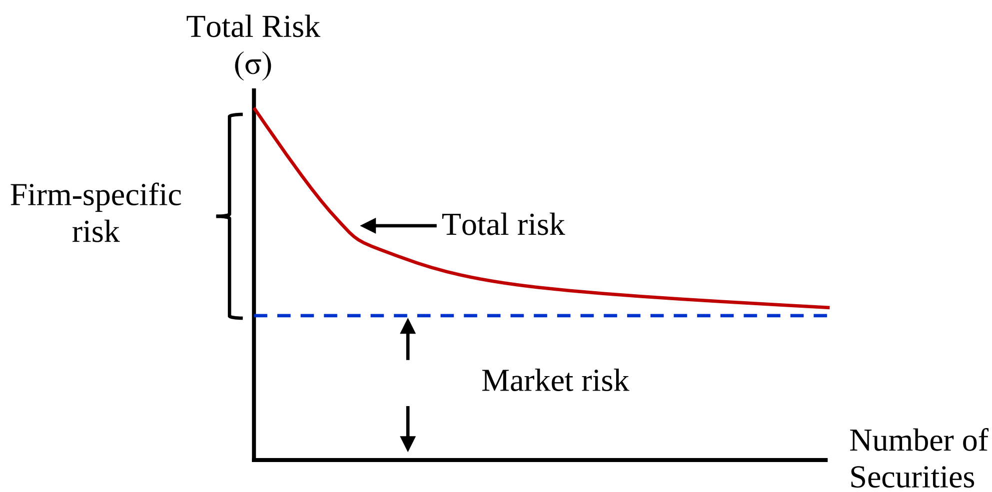
EFFICIENT DIVERSIFICATION

Diversification and Portfolio Risk

- Systematic versus unsystematic risk
    - Systematic risk—market risk that cannot be reduced through diversification; risk associated with the nation's economy; also referred to as market risk, economic risk, and nondiversifiable risk.
    - Unsystematic risk—risk unique to the firm/industry that can be reduced (theoretically to zero) through diversification; also referred to as firm-specific risk, nonsystematic risk, and unsystematic risk.
    - The following graph shows the effects of diversification:



- Asset Allocation with Two Risky Assets—the relationship between two risky investments is important when combining them to form a two-asset portfolio.
    - Portfolio return—weighted average of the expected returns of the investments that make up the portfolio.

$$E(r_P) = \sum_{i=1}^{n} w_i E(r_i)$$

For a two-asset portfolio:

$$E(r_P) = w_A E(r_A) + w_B E(r_B)$$

Here, w_A represents the proportion of total funds invested in Investment A; $w_B = 1 - w_A$.

- Covariance and correlation—provides an indication as to how two investments are related
    - Covariance is a measure of the co-movement (positive, negative, or none) of the deviations of two assets' returns; provides an indication as to whether the returns of two investment have a tendency to move together, have a tendency to move opposite each other, or have no relationship (pattern) at all; gives the "direction" of the relationship.

$$\text{Cov}(r_A, r_B) = \sigma_{A,B} = \sum_{i=1}^{n} Pr_i[r_{A,i} - E(r_B)][r_{B,i} - E(r_B)]$$

Pr_i is the probability that Event i will occur, $r_{A,i}$ is the payoff for Investment A when Event i occurs, and $E(r_A)$ is the expected return for Investment A.

- Correlation—standardization of covariance; provides an indication of both direction and strength of assets' relationships

$$\rho_{A,B} = \frac{\text{Cov}(r_A, r_B)}{\sigma_A \sigma_B}$$

- Portfolio variance
  - Direct method—compute portfolio returns for ***each*** "state of nature," then compute the variance as a deviation from the expected value of the portfolio

$$\sigma_P^2 = \sum_{i=1}^{n} Pr_i[r_{P_i} - E(r_P)]^2$$

  - Indirect method—covariance must be computed

    2-asset portfolio:

$$\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \text{Cov}(r_A, r_B)$$

$$= w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B (\rho_{A,B}\sigma_A\sigma_B)$$

    3-asset portfolio:

$$\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2$$

$$+2w_A w_B \text{Cov}(r_A, r_B) + 2w_A w_C \text{Cov}(r_A, r_C) + 2w_B w_C \text{Cov}(r_B, r_{CB})$$

- Efficient and inefficient investments
  - Mean-variance criterion (frontier)
    - The **minimum variance set** is represented by the set of investments that produce the least risk (variance) at a given level of return. Such investments might not be efficient according to the definition given below.
    - The **efficient set** of investments represents those positions that promise the greatest return at a given level of risk (variance). Other positions are considered **inefficient** either due to risk (higher), return (lower), or both.
    - Minimum variance portfolio (MVP)—the combination of assets that gives the lowest possible **total risk** (variance). For a two-asset portfolio the weights necessary to achieve minimum variance can be computed as follows:

$$w_A = \frac{\sigma_B^2 - \text{Cov}(r_A, r_B)}{\sigma_A^2 + \sigma_{B-2}^2 - 2\text{Cov}(r_A, r_B)} = \frac{\sigma_B^2 - \rho_{A,B}\sigma_A\sigma_B}{\sigma_A^2 + \sigma_{B-2}^2 - 2(\rho_{A,B}\sigma_A\sigma_B)}$$

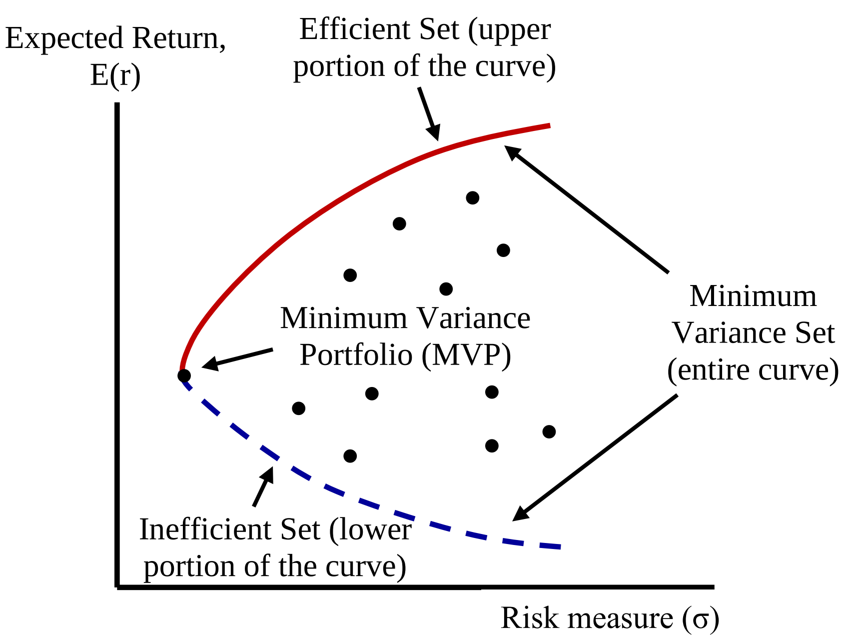
- When $\rho = 0$, the equation simplifies to:

$$w_A = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}$$

- When $\rho = -1$, the equation simplifies to:

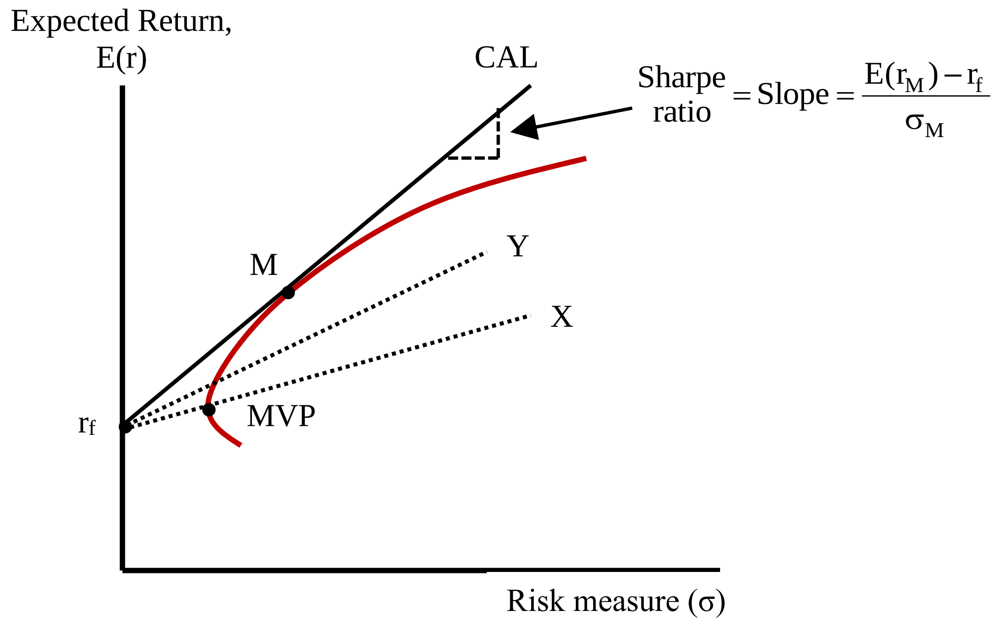
$$w_A = \frac{\sigma_B}{\sigma_A + \sigma_B}$$

- Graph of opportunity set, minimum variance set, and efficient set—the opportunity set includes all investments on the curve, even the dashed curve, and all of the investments (points) inside the curve:



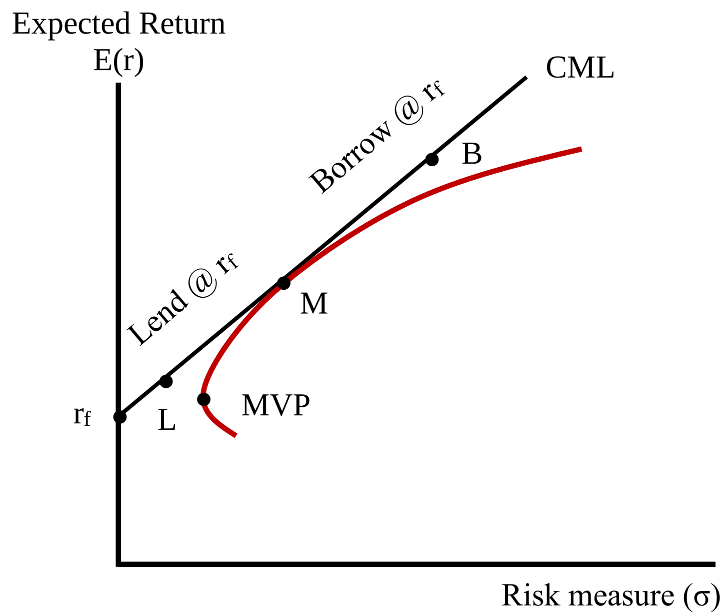
- The ***efficient set*** represents the best portfolios of risky investments based on risk and return. These portfolios have the greatest return for a given level of risk. As a result, the curve that is highlighted in red represents the set of feasible risky investments for rationale investors.
- The ***inefficient set*** represents the portfolios of risky investments that have the lowest return for a given level of risk. Although these portfolios have the lowest risk for a particular return, they are not efficient because portfolios with greater returns exist for the specific level of risk. As a result, investors should not invest in risky portfolios that are on the curve that is highlighted with the blue dashed line.

- Combining portfolios on the efficient set with a risk-free asset—if we include a risk-free asset—e.g., a Treasury bill—we find that investors can form complete portfolios that include a risky asset (i.e., a risky portfolio on the efficient set) and a risk-free asset, which looks like the following:



Here, investors can borrow and lend at the risk-free rate (r_F).

- Investors would not invest along either Line X, which goes through the minimum variance portfolio (MVP), or line Y, because they could attain a better Sharpe ratio by investing along the line that has the steepest slope (the highest Sharpe ratio). The line with the highest Sharpe ratio is the capital allocation line (CAL). All rationale investors will select portfolios along the CAL.
- Separation theorem (property)—according to the previous explanation, every investor will invest in only two assets: (1) the risk-free asset and (2) the risky portfolio, which includes all available risky investments. The choice that investors make is whether to invest positive amounts in both the risk-free asset and the risky asset, whether to invest 100 percent of their funds in the risky asset only, or whether to borrow money so that they can invest more than 100 percent of their funds in the risky asset. Thus, the particular investments every investor should select are determined by the mean-variance criterion; that is, all investors should select the same two investments—the risk-free asset and the risky asset. The optimal risky asset is the same for all investors. Investors decide what combination of the risk-free asset and the risky asset they prefer to hold, which is based on their risk preferences. As a result, the choice between what assets should be included in a portfolio and how much to allocate to each investment (i.e., the financing decision) are *separate*, independent decisions. Stated differently, regardless of their risk preferences, to maximize utility, ***all*** investors will invest in the risk-free asset and a portfolio that contains all the risky assets in the market. How much will be invested in each position depends on the risk attitude (preference) of each investor. Thus, the investment decision (what assets to invest in) is separated from the financing decision (the proportion of each that should be held—i.e., how much to borrow or lend). The following graph shows this relationship:



If investors can construct portfolios at no cost (commissions, etc.) using any or all of the assets available in the market, the line in the graph is called the capital market line and its equation is:

$$E(r_I) = r_f + \left[\frac{E(r_M) - r_f}{\sigma_M}\right]\sigma_I$$

where σ_I is the standard deviation of a particular investor's investment position.

- Portfolio L represents a portfolio in which 75 percent of the total funds is invested in the risk-free asset and 25 percent is invested in the risky asset. Suppose the risk-free asset has an expected return equal to 4 percent and the optimal risky portfolio has an expected return equal to 12 percent and a standard deviation equal to 20 percent. Portfolio L's expected return and standard deviation are:

  $$E(r_L) = 0.75(4\%) + 0.25(12\%) = 6\%$$

  $$\sigma = 0.25(20\%) = 5\%$$

- Portfolio B represents a portfolio in which 180 percent of the total funds is invested in the risky asset, where the additional 80 percent comes from a loan at the risk-free rate (i.e., the risk-free asset is shorted). Portfolio B's expected return and standard deviation are:

  $$E(r_M) = -0.8(4\%) + 1.8(12\%) = 18.4\%$$

  $$\sigma = 1.8(20\%) = 36\%$$

- Single-index stock market—because the market is perfectly diversified and moves only due to systematic risk factors, a market index model can be used to divide total risk into systematic risk and firm-specific risk. For example, if $R_i = r_i - r_f$, which is the excess return for Security i, we can use the following equation to differentiate between systematic risk components and firm-specific risk components:

$R_i = \alpha_i + \beta_i R_M + e_i$

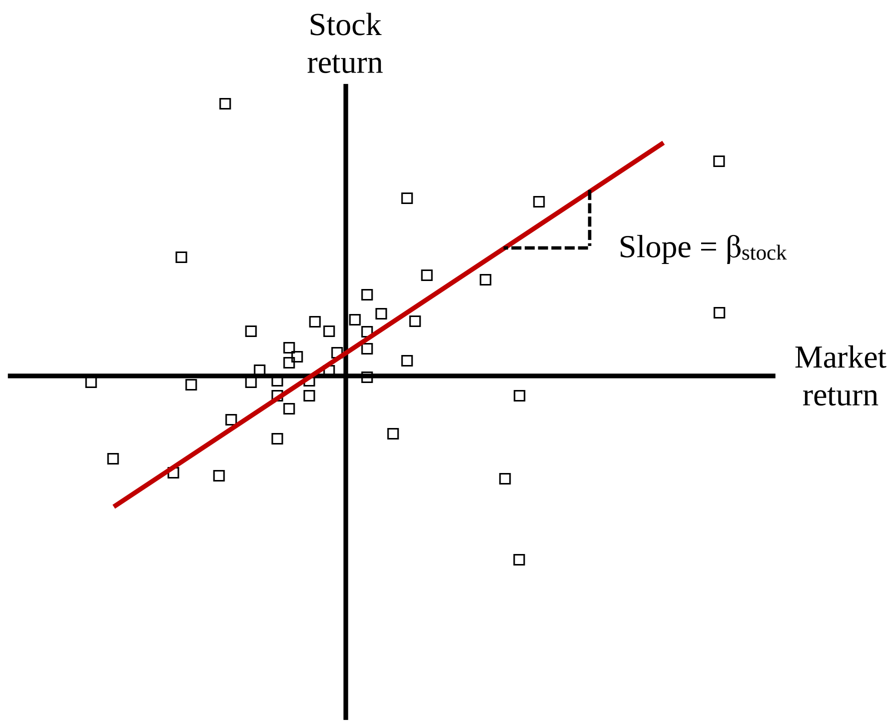
Here, R_M represents the excess return on the market, which should only be affected by systematic (economic) factors; β_i provides an indication of the relationship between the stock's excess return and the market's excess return, which is a measure of the stock's systematic risk; e_i shows the relationship between the stock's excess returns and non-economic factors, which represent the firm-specific risk associated with the stock; and α_i indicates whether the stock is overvalued (+ number) or undervalued (– number).

Because there should be no relationship between market returns and firm-specific returns, the variance of R_i can be written:

$$\sigma^2_{R_i} = \sigma^2(\alpha_i + \beta_i R_M + e_i)$$

$$\begin{aligned} &= \sigma^2(\beta_i R_M) \quad + \quad \sigma^2(e_i) \\ &= \beta_i^2 \sigma_M^2 \quad + \quad \sigma^2(e_i) \\ &= \text{Systematic risk} + \text{Firm-specific risk} \end{aligned}$$

- Using the single-index model to determine an investment's systematic risk—plot the historical returns on a stock with the historical returns on the market and perform a simple regression analysis to determine the slope of the line that best fits through the plotted points, For example, consider the following:



The equation for this line is $E(r_{stock}) = \alpha + E(r_M)\beta_{stock} + e_{stock}$. In this form, the intercept, α, should equal zero when the stock is correctly valued. The returns can be stated either as actual returns as indicated above or as excess returns. If excess returns are used, the equation is essentially the same, except the intercept term will equal the risk-free rate of return. In this case the equation can be written as:

$$E(r_{stock}) = r_f + R_M\beta_{stock} + e_{stock}$$

where R_M is the excess return on the market (i.e., the market's risk premium). The term e_{stock} represents the stock's firm-specific risk, which is depicted on the graph as the points that do not fall on the regression line. Points do not fall on the regression line due to firm-specific events (risk) that do not arise as the result of economic movements.

The beta coefficient, β, indicates by how much the stock's return normally moves when the market return moves by 1 percent.

- Total risk versus systematic risk—the proportion of the total risk associated with an investment can be determined as follows:

$$\begin{matrix}\text{Explained} \\ \text{variation}\end{matrix} = \rho^2 = \frac{\text{Systematic risk}}{\text{Total risk}} = \frac{\beta^2_{stock}\sigma^2_M}{\sigma^2_{stock}} = \frac{\beta^2_{stock}\sigma^2_M}{(\beta^2_{stock}\sigma^2_M) + \sigma^2(e_{stock})}$$

If $\rho^2 = 0.6$, then 60 percent of the movements in the stock's returns is explained by movements in the market's returns, which means 60 percent of the stock's total risk is comprised of systematic risk and 40 percent is comprised of firm-specific risk.