## Managing Bond Portfolios

## Interest Rate Risk

- Interest rate risk is the risk market rates will change.
- Price risk—bond prices (values) move opposite interest rates; there exists an inverse relationship such that prices decrease when market rates increase, and vice versa.
- Reinvestment risk -interest rate changes are positively related to the ability to reinvest at favorable rates; that is, when market rates increase, the coupon interest that is paid by a bond can be reinvested at higher rates, and vice versa.
- Interest rate sensitivity-the characteristics of bonds indicate how sensitive bond prices are to changes in interest rates.
- Bond prices move opposite changes in interest rates (inversely related).
- When market yields increase, bond price decrease, and vice versa.
- A price decrease that results from an increase in the market rate will be less than a price increase that results from an equivalent decrease in the market rate.
- Everything else equal, bond prices are more sensitive for bonds with:
- Longer terms to maturity than bonds with shorter terms to maturity.
- The sensitivity of longer-term bonds increases at a decreasing rate, which means that a bond with 20 years to maturity is not twice as sensitive to changes in market rates than a bond with 10 years to maturity; e.g., the price of a 10 -year bond might decrease by 6 percent when market rates increase, whereas the price of the 20year bond would decrease by 9 percent for the same change in market rates.
- Bonds with lower coupon rates than bonds with higher coupon rates.
- Bonds with lower yields to maturity than bonds with higher yields to maturity.
- Example: Consider the three bonds with the characteristics given in the following table. Interest is paid annually.
- Bond A and Bond B are identical, except the years to maturity is greater for Bond B than for Bond A . When the market rate changes from 5 percent to 7 percent, Bond B's price changes by a greater percentage than Bond A's price changes.
- Bond A and Bond C are identical, except the coupon rate is lower for Bond A than for Bond C. When the market rate changes from 5 percent to 7 percent, Bond A's price changes by a greater percentage than Bond C's price changes.

|  | Bond A | Bond B | Bond C |
| :---: | :---: | :---: | :---: |
| Years to maturity | 10 yrs | 20 yrs | 10 yrs |
| Coupon rate | 4.0\% | 4.0\% | 6.0\% |
| Face value | \$1,000 | \$1,000 | \$1,000 |
| Yield | Bond A | Bond B | Bond C |
| 5\% | \$922.78 | \$875.38 | \$1,077.22 |
| 7\% | \$789.29 | \$682.18 | \$929.76 |
| \$ $\Delta$ | (\$133.49) | (\$193.20) | (\$147.45) |
| \% $\Delta$ | -14.5\% | -22.1\% | -13.7\% |

- Suppose Bond D has the same characteristics as Bond A, except its current yield to maturity is 8 percent. Bond D's current price would be $\$ 731.60$. If the market rate on Bond D increases by 2 percentage points, from 8 percent to 10 percent, its value will decrease to $\$ 631.33$, which is a decrease of 13.7 percent. The decrease in Bond D's price is less than the decrease in Bond A's price when its yield increased 2 percentage points.
- Duration-measures a bond's "effective" maturity
- Represents the time it takes an investment to repay the amount initially invested plus the return expected from the investment.
- If a bond is held for the length of its duration, the investor should earn the return expected when the bond was purchased, even if market rates change (this only applies for one interest rate change)
- Duration is the length of time it takes to offset price risk with reinvestment risk when market rates change.
- Macaulay's duration

$$
\left.\begin{array}{rl}
\text { DUR } & \left.=\frac{\left\{(1) \times\left[\frac{\mathrm{CF}_{1}}{(1+\mathrm{y})^{1}}\right]\right\}+\left\{(2) \times\left[\frac{\mathrm{CF}_{2}}{(1+\mathrm{y})^{2}}\right]\right\}+\left\{(3) \times\left[\frac{\mathrm{CF}_{3}}{(1+\mathrm{y})^{3}}\right]\right\}+\cdots+\left\{(\mathrm{n}) \times\left[\frac{\mathrm{CF}_{\mathrm{n}}}{(1+\mathrm{y})^{\mathrm{n}}}\right]\right\}}{\left[\frac{\mathrm{CF}}{1}\right.} \overline{(1+\mathrm{y})^{1}}\right]+\left[\frac{\mathrm{CF}_{2}}{(1+\mathrm{y})^{2}}\right]+\left[\frac{\mathrm{CF}_{3}}{(1+\mathrm{y})^{3}}\right]+\cdots+\left[\frac{\mathrm{CF}}{\mathrm{n}}\right. \\
(1+\mathrm{y})^{\mathrm{n}}
\end{array}\right] \quad\left(\begin{array}{l}
\sum_{\mathrm{t}=1}^{\mathrm{n}}\left\{(\mathrm{t}) \times\left[\frac{\mathrm{CF}}{(1+\mathrm{y})^{\mathrm{t}}}\right]\right\} \\
\end{array}\right.
$$

Here, $\mathrm{CF}_{\mathrm{t}}=$ the cash flow that occurs in Period t ; $\mathrm{y}=$ the market yield per period, which is YTM/2 when interest is paid semiannually; $\mathrm{P}_{0}=$ current price of the bond; and $\mathrm{w}_{\mathrm{t}}=$ the weight of the present value of the cash flow generated in Period $t$ stated as a percent of the bond's price, which is

$$
\left.\left.\mathrm{w}_{\mathrm{t}}=\frac{\left[\frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{y})^{\mathrm{t}}}\right]}{\mathrm{P}_{0}}=\frac{\left[\frac{\mathrm{CF}_{\mathrm{t}}}{(1+\mathrm{y})^{\mathrm{t}}}\right]}{\left[\frac{\mathrm{CF}}{1}\right.} \overline{(1+\mathrm{y})^{1}}\right]+\left[\frac{\mathrm{CF}_{2}}{(1+\mathrm{y})^{2}}\right]+\left[\frac{\mathrm{CF}_{3}}{(1+\mathrm{y})^{3}}\right]+\cdots+\left[\frac{\mathrm{CF}_{\mathrm{n}}}{(1+\mathrm{y})^{\mathrm{n}}}\right]\right]
$$

The DUR equation can be simplified to:

$$
\operatorname{DUR}_{\mathrm{n}}=\left(\frac{1+\mathrm{y}}{\mathrm{y}}\right)-\left[\frac{(1+\mathrm{y})+\left(\mathrm{n} \times\left\{\frac{\mathrm{C}}{\mathrm{~m}}-\mathrm{y}\right\}\right)}{\left\{\frac{\mathrm{C}}{\mathrm{~m}} \times\left[(1+\mathrm{y})^{\mathrm{n}}-1\right]+\mathrm{y}\right\}}\right]
$$

## Variable Definitions

$\mathrm{m}=$ the number of interest payments per year
$\mathrm{n}=$ number of interest payments that remain to maturity
$\mathrm{y}=$ yield per compounding period $=\mathrm{YTM} / \mathrm{m}$
$\mathrm{C}=$ annual coupon rate of interest for a bond stated in decimal form

- Duration example-consider the following bond, which has a $\$ 1,000$ face value, matures in eight (8) years, and has a 5 percent coupon rate of interest (interest is paid annually). The current market yield is 6 percent. The table shows how to compute the bond's duration using the first equation provided in the previous section.

| Period <br> $($ year $)$ | Cash Flow <br> $(\mathrm{CF})$ | PV of $\mathrm{CF}_{t}$ <br> at $\mathrm{YTM}=6 \%$ | Weight $\left(\mathrm{w}_{\mathrm{t}}\right)$ <br> $=\left(\mathrm{PV}\right.$ of $\left.\mathrm{CF}_{t}\right) / \mathrm{P}_{0}$ | DUR <br> Period $\mathrm{x} \mathrm{w}_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 50.00$ | $\$ 47.1698$ | 0.0503 | 0.0503 |
| 2 | 50.00 | 44.4998 | 0.0474 | 0.0949 |
| 3 | 50.00 | 41.9810 | 0.0448 | 0.1343 |
| 4 | 50.00 | 39.6047 | 0.0422 | 0.1689 |
| 5 | 50.00 | 37.3629 | 0.0398 | 0.1992 |
| 6 | 50.00 | 35.2480 | 0.0376 | 0.2255 |
| 7 | 50.00 | 33.2529 | 0.0355 | 0.2482 |
| 8 | $1,050.00$ | $\underline{658.7830}$ | $\underline{0.7024}$ | $\underline{5.6192}$ |
|  |  | $\mathrm{P}_{0}=\mathbf{9 3 7 . 9 0 2 1}$ | $\underline{1.0000}$ | $\mathbf{6 . 7 4 0 4}=$ DUR |

Using the simplified equation to compute duration gives:

$$
\begin{aligned}
\operatorname{DUR}_{\mathrm{n}} & =\left(\frac{1.06}{0.06}\right)-\left[\frac{(1.06)+\left(8 \times\left\{\frac{0.05}{1}-0.06\right\}\right)}{\left\{\frac{0.05}{1} \times\left[(1.06)^{8}-1\right]+0.06\right\}}\right] \\
& =17.6667-10.9262=6.7404 \text { years }
\end{aligned}
$$

A duration equal to 6.74 years indicates that it will take 6.74 years for an investor to
recapture the $\$ 937.90$ cost of the bond plus a 6 percent return on the investment.

- Duration can be used to estimate the change in the price of a bond that will occur when the market rate changes.

$$
\frac{\Delta \mathrm{P}}{\mathrm{P}}=\% \Delta \mathrm{P} \approx-\operatorname{DUR}\left[\frac{\left(1+\mathrm{y}_{1}\right)-\left(1+\mathrm{y}_{0}\right)}{\left(1+\mathrm{y}_{0}\right)}\right]=-\operatorname{DUR}\left[\frac{\mathrm{y}_{1}-\mathrm{y}_{0}}{1+\mathrm{y}_{0}}\right]
$$

This relationship can also be stated as:

$$
\frac{\Delta \mathrm{P}}{\mathrm{P}}=\% \Delta \mathrm{P} \approx-\mathrm{D} * \Delta \mathrm{y}=-\mathrm{D} *\left(\mathrm{y}_{1}-\mathrm{y}_{0}\right)
$$

where $\mathrm{D}^{*}=\frac{\text { DUR }}{1+\mathrm{y}_{0}}=$ Modified duration
For example, if these equations are applied to bond described in the duration example, the approximate change in price that would be expected when the market rate increases from 6 percent to 7 percent is:

$$
\% \Delta \mathrm{P} \approx-6.74\left[\frac{0.07-0.06}{1.06}\right]=-0.0636=-6.36 \%
$$

The price of the bond when the market rate is 7 percent is $\$ 880.57$. As a result, the percent change in price when the rate increases from 6 percent to 7 percent is:

$$
\% \Delta \mathrm{P}=\frac{\$ 880.57-\$ 937.90}{\$ 937.90}=-0.0611=-6.11 \%
$$

- Determinants of duration-the characteristics of a bond relative to the market rate.
- The duration of a zero-coupon bond is its term to maturity.
- If two bonds are identical except for their coupon rates, the bond with the lower coupon rate will have a higher duration.
- If two bonds are identical except for their terms to maturity, the bond with the greater time to maturity will have a greater duration.
- If two bonds are identical except for their yields to maturity, the bond with the lower yield to maturity will have a greater duration.
- These relationships are the same as was stated earlier for the sensitivity of a bond's price when market yields change. That is, bonds are more sensitive when their coupon rates are lower, when their yields to maturity are lower, and when their terms to maturity are higher. These same bonds would also have higher durations.


## Passive Bond Management

- Passive managers primarily attempt to control the risk of a bond portfolio.
- Immunization is a strategy that is used to try to reduce or eliminate the effects of interest rate risk; that is, the effects of changes in interest rates.
- Immunization can be used to offset the change in price with the change in reinvestment income that results from a change in market interest rates.
- Immunization attempts to offset price risk with reinvestment risk by matching duration.
- If a bond (or a bond portfolio) has a duration equal to the investment horizon of an investor, the investor's position is considered immunized in the sense that the investor's total expected return should not be affected by a change in market interest rates. A loss (gain) of return that results from a price decline (increase) will be offset by a gain (loss) of return that results from the ability to reinvest coupon interest payments at higher (lower) rates.
- Immunization is good only for one change in interest rates. After a change in market rates, the bond (or bond portfolio) must be re-immunized (rebalanced or realigned).
- Immunization example: The following bonds have a coupon rate equal to 5 percent. Interest is paid annually.

Bond Characteristics:

|  | Bond A  Bond B  <br> Bond C    <br> Maturity ( $\mathrm{n}=$ years) 8.0  10.0 <br>  $\$ 937.90$  $\$ 926.40$ <br>  $\$ 916.16$   <br> Price today 6.74  8.02 <br>   9.17  <br> Duration—years $6.00 \%$  $6.00 \%$ | $6.00 \%$ |
| :--- | ---: | ---: | ---: | ---: |

- If an individual wishes to invest for a period of eight (8) years, he or she would be immunized from an interest rate change if Bond $B$ is purchased, because Bond $B$ has a duration that equals the investment horizon (period).
- Suppose the market rate decreases to 4 percent immediately after the investor purchases Bond B, and the rate remains at 4 percent for the remainder of the investment period (i.e., for the next eight years the market interest rate is 4 percent).
- The following table shows the payoffs that would exist for each of the three bonds at the end of the eight-year investment horizon (i.e., eight years from now).

Scenario 1-the market rate drops from 6 percent to 4 percent, and then remains at 4 percent for the eight-year investment period.

|  | $\underline{\text { Bond A }}$ | $\underline{\text { Bond B }}$ | $\underline{\underline{\text { Bond C }}}$ |
| :--- | :---: | :---: | :---: |
| Purchase price—Year 0 | $\$ 937.90$ | $\$ 926.40$ | $\$ 916.16$ |
| Remaining maturity—years | 0.0 | 2.0 | 4.0 |
| Price in Year 8 | $\$ 1,000.00$ | $\$ 1,018.86$ | $\$ 1,036.30$ |
| Dollar coupon interest <br> received over eight years | 400.00 | 400.00 | 400.00 |
| Interest (return) earned from <br> reinvesting dollar coupon interest | $\underline{60.71}$ | $\underline{60.71}$ | $\underline{60.71}$ |

Total wealth in Year 8
Effective yield
As you can see, Bond B earned nearly the same return that was expected when it was purchased eight years ago, which was 6.0 percent. On the other hand, Bond A earned less than the expected return and Bond C earned more than the expected return.

- To see what caused the return on Bond A to be less than expected and the return on Bond C to exceed expectations, let's examine what would have happened if the market rate had not changed immediately after Bond B was purchased; that is, the rate remained at 6 percent for the eight-year investment horizon. In this case, the payoffs in eight years would be:
Scenario 2-the market rate remains at the original 6 percent for the entire eight-year investment period.

|  | $\underline{\text { Bond A }}$ | $\underline{\text { Bond B }}$ | $\underline{\text { Bond C }}$ |
| :--- | :---: | :---: | :---: |
| Purchase price—Year 0 | $\$ 937.90$ | $\$ 926.40$ | $\$ 916.16$ |
| Remaining maturity—years | 0.0 | 2.0 | 4.0 |
| Price in Year 8 | $\$ 1,000.00$ | $\$ 981.67$ | $\$ 965.35$ |
| Dollar coupon interest <br> received over eight years <br> Interest (return) earned from <br> reinvesting dollar coupon interest | $\mathbf{\$ 0 0 . 0 0}$ | 400.00 | 400.00 |
| Total wealth in Year 8 |  |  |  |

- Comparing the cash flows from the two previous tables, we have:

|  | Bond A | Bond B | Bond C |
| :---: | :---: | :---: | :---: |
| Price in Year 8 |  |  |  |
| Scenario 1 | \$1,000.00 | \$1,018.86 | \$1,036.30 |
| Scenario 2 | 1,000.00 | 981.67 | 965.35 |
| Difference in price | \$0.00 | \$37.19 | \$70.95 |
| Reinvestment interest |  |  |  |
| Scenario 1 | \$60.71 | \$60.71 | \$60.71 |
| Scenario 2 | 94.87 | 94.87 | 94.87 |
| Difference in price | \$(34.16) | \$(34.16) | \$(34.16) |
| Gain (loss) from rate $\Delta$ | \$(34.16) | \$3.03 | \$36.79 |

You can see that reinvestment risk dominated Bond A , which had a duration lower than the investment horizon, whereas price risk dominated Bond C , which had a duration higher than the investment horizon.

- After the market rate change in Scenario 1, the duration for each bond changes. After the rate change from 6 percent to 4 percent, the durations would be:
Duration $\quad \frac{\text { Bond A }}{6.83 \mathrm{yrs}} \quad \frac{\text { Bond B }}{8.19 \mathrm{yrs}} \quad \frac{\text { Bond C }}{9.44 \mathrm{yrs}}$

You can see that the decrease in the market rate caused the durations to increase. The reason for this result is because the time to recover the initial investment increases when interest rates decline. The opposite effect occurs when market interest rates increase.

- Immunization can be used to attain cash flow matching, which is when the cash flows from a bond matches an obligation that must be paid in the future. Such a strategy must be rebalanced after every significant change in market rates.
- Dedication strategy is the matching of cash flows on a multiperiod basis. Such a strategy entails immunizing a portfolio of bonds by purchasing individual bonds (or other investments) with maturities that match the dates obligations must be paid. There is no need to rebalance such a portfolio because the maturity values of the bonds are known and do not change when market interest rates change.
- Portfolio duration- $\mathrm{DUR}_{\mathrm{P}}$ is the weighted average of the durations for the individual bonds included in the portfolio.

$$
\operatorname{DUR}_{\mathrm{P}}=\mathrm{w}_{1} \mathrm{DUR}_{1}+\mathrm{w}_{2} \mathrm{DUR}_{2}+\cdots+\mathrm{w}_{\mathrm{n}} \mathrm{DUR}_{\mathrm{n}}=\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{w}_{\mathrm{j}} \times \mathrm{DUR}_{\mathrm{j}}
$$

where $w_{j}$ is the proportion of total funds invested in Bond $j$ and $D U R_{j}$ is Bond $j$ 's duration. It is easier to immunize an investment position with a bond portfolio than to find a single bond that will immunize the position.

## Convexity

- Convexity refers to the curvature of the graph that shows the relationship between the change in yield and the change in the price of a bond; higher convexity indicates greater curvature.
- Investors like convexity because bonds with great convexity provide a greater increase in price when market rates fall than the decrease that occurs when market rates decline by the same amount.
- As the result of convexity, the equation used to approximate the effect of a change in the market rate on the price of a bond provide a good estimate only when the rate change is not substantial. This occurs because the approximation equation given earlier represents a linear relationship (would plot as a line), whereas the actual relationship between rate changes and price changes is convex (i.e., curved). The equation given earlier can be modified to provide a better estimation:

$$
\frac{\Delta \mathrm{P}}{\mathrm{P}}=\% \Delta \mathrm{P} \approx-\mathrm{D} * \Delta \mathrm{y}+1 / 2\left[\text { Convexity } \times(\Delta \mathrm{y})^{2}\right]
$$

## Active Bond Management

- Interest rate forecasting-if market rates are expected to decrease, bond managers will increase the portfolio's duration, and vice versa.
- Identifying mispriced bonds-investing in mispriced bonds can lead to abnormal returns. It is not easy to find mispriced bonds, however.
- Bond swaps used by bond portfolio manager-exchange, or swap, bonds with another bond manager to attain higher expected returns.
- Substitution swaps-swap one bond with an identical bond that is thought to be mispriced.
- Intermarket swap-swap bonds from different sectors of the bond market; believe the spread between yields is not normal.
- Rate anticipation swap-swap bonds because interest rates are forecasted to change; e.g., if interest rates are forecasted to decrease, swap bonds to make the duration of the portfolio higher (longer term to maturity).
- Pure yield pickup swap-try to increase yield by swapping lower yielding bonds with higher yielding bonds; investor is willing to take the interest rate risk.
- Horizon analysis-investment strategy is based on predictions of the market rate that will exist at the end of the investment horizon. Reinvestment income and capital gains are computed based on the forecasted rate.

